Fuzzy rule-based systems are systems that employ a certain inference mechanism deducing conclusions based on an appropriate model of a fuzzy rule base that is a set of fuzzy IF-THEN rules. These systems became very popular mainly due the first successful industrial experiment by E. H. Mamdani and S. Assilian. It is worth noticing that the authors used so called ‘conjunctive’ fuzzy model of a fuzzy rule base, i.e., a fuzzy relation that does not employ any kind of implication that would be naturally expected in the model by mathematicians and logicians. Success of the implementation elicited numerous followers and the conjunctive models became very popular among the community of practitioners. The popularity of these models nearly eliminated any interest of practitioners in the models that directly employ fuzzy implications – the ‘implicative’ fuzzy models. This becomes more interesting having in mind that even Lotfi Zadeh already in 1973 (two years before the publication of E. H. Mamdani and S. Assilian) recalled the truth table of classical implication and recalled the fact that in classical logic the following holds

\[ a \Rightarrow b \equiv \neg a \lor b. \]

Moreover, only three years after the successful experiment, i.e., in 1978, Lotfi Zadeh represented a single fuzzy IF-THEN rule with help of the Łukasiewicz fuzzy implication.

Logicians who originally did not share the enthusiasm for conjunctive fuzzy models, later on showed that both implicative and conjunctive models are logically correct. However, it is important to stress that they have to be understood and treated in a different way.

In this lecture, we do not intend to contribute too much to the logical, philosophical and interpretation aspects of the discussion on these two basic families of models of fuzzy rule bases. The goal of this contribution is totally different – in particular, to provide an insight into the most fundamental mathematical properties of such systems in order to find an appropriate combination of the “bricks” we use to built a fuzzy rule-based system in order to preserve the desirable properties. Therefore, we will for example present how to combine a fuzzy-rule based model (either implicative or conjunctive) with an appropriate inference mechanism and an appropriate fuzzy partition of the input space in
order to preserve the modus ponens property or, how to combine the fuzzy rule based model with an appropriate defuzzification and appropriate antecedent as well as consequent fuzzy sets in order to preserve the monotonicity property. We will also address the question of mutually exclusivity of such properties or their possible co-existence. Finally, in order to extend the survey, we will present also some alternative approaches to modeling the fuzzy rule-based systems that are not that universally known however, they surely deserve some attention.